Goniometrické vzorčeky

$$1 = \sin^2 x + \cos^2 x$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin x \pm \sin y = 2\sin\frac{x \pm y}{2}\cos\frac{x \mp y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2} \qquad \cos^2\frac{x}{2} = \frac{1+\cos x}{2}$$

$$\sin(\arccos x) = \sqrt{1-x^2} \quad \Leftarrow \quad \sin x = \sqrt{1-\cos^2 x}$$

$$\cos(\arcsin x) = \sqrt{1-x^2} \quad \Leftarrow \quad \cos x = \sqrt{1-\sin^2 x}$$

$$\sin(\arctan x) = \frac{x}{\sqrt{1+x^2}} \quad \Leftarrow \quad \sin x = \frac{\tan x}{\sqrt{1+\tan^2 x}}$$

$$\cos(\arctan x) = \frac{1}{\sqrt{1+x^2}} \quad \Leftarrow \quad \cos x = \frac{1}{\sqrt{1+\tan^2 x}}$$

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

Limity

$$\lim_{x\to\infty}\frac{\sin x}{x} = 1$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e \iff \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$$

$$\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1 \qquad \lim_{x\to 0}\frac{e^x - 1}{x} = 1$$

Integrály

$$\int f(x)\ dx = F(x) + c$$

$$\int \frac{f'(x)}{f(x)} = \ln|f(x)|$$

$$\int x^n\ dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{x}\ dx = \ln|x| + c$$

$$\int e^x\ dx = e^x + c$$

$$\int a^x\ dx = \frac{a^x}{\ln a} + c\ (a > 0, a \neq 1)$$

$$\int \frac{1}{1+x^2}\ dx = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + c;$$

$$\int \frac{1}{\sqrt{1-x^2}}\ dx = \arcsin x + c = -\arccos x + c$$

$$\int \sin x\, dx = -\cos x + c \qquad \int \sinh x\, dx = \cosh x + c$$

$$\int \cos x\, dx = \sin x + c \qquad \int \cosh x\, dx = \sinh x + c$$

pre $a > 0$

$$\int \frac{1}{a^2+x^2}\ dx = \frac{1}{a}\arctan\frac{x}{a} + c$$

$$\int \frac{1}{a^2-x^2}\ dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$$

$$\int \frac{1}{\sqrt{a^2+x^2}}\ dx = \arcsin\frac{x}{a} + c$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\ dx = \ln|x + \sqrt{x^2 \pm a^2}| + c$$

per partes: $\int u'(x)v(x)dx = u(x)v(x) - \int u(x)v'(x)dx$